Art. XVI.—*On the Ratio of the Length and Height of Sea Waves.*

By S. R. Deverell, Esq.

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Of the phenomena appertaining to water-waves none seem to have appeared more capricious to observers than the variable proportion of the height to the length of waves. Indeed such strange diversities are exhibited in this respect that writers have used themselves to speak of different kinds of waves as if they were of different species:—The short chopping sea; the steep high sea; the long high sea; the long roll, of medium height and length—that measured tread of old ocean, as an Arctic voyager has expressed it, which so gladdens the eyes and the heart of the Polar sojourner when he first strikes it; finally, the tremendous "comber" of navigators which from overhead threatens to bury the ship: these are often referred to as originating rather from different causes than as being so many transitions or attitudes of the same thing or entity. There is, again, the mysterious ground swell, which old seamen firmly believe to arise in some occult manner from the bottom, proceeding in slow, languid oscillations, but breaking with an everlasting roar and violence on the shore to which it is bound. Mere magnitude does not appear to be an essential characteristic of any of these forms, for they may all be met with in various degrees of size. Scoresby mentions waves in the Southern Ocean a quarter of a mile from peak to peak; but this can be by no means unusual, for in that vast sea, which may in truth be said to be the native home of the great waves, five waves to a mile is a very ordinary occurrence in a westerly gale, and the writer has counted five to a mile when the waves have not been more than six or seven feet high. The length of a wave in fact is by no means a criterion of its height: its actual magnitude is rather measurable by the area of a vertical length-section than by the height. Again, as regards the speed, the velocity, says Mr. Reed, seems to depend almost entirely on the length of a wave and not at all upon the height. It should be remembered that existing knowledge on these subjects, to which general attention has only been attracted during a few years past, is at present in an immature or rather embryonic state, as indeed is continually pointed out by its most eminent followers. The views and suggestions of any observer, however humble, are of value; and the store of information which the British and French Admiralties—ever rivals in scientific progress—are now engaged in collecting, through their naval officers, in all parts of the world, must soon tend to formulate a completed theory of the subject. The extraordinary length of some waves in comparison with their height has often attracted the notice and the vague surprise of observers long even before the attention of mathematicians was drawn into the inquiry. In a recent Admiralty circular Mr. Froude cautions officers observing waves that they must not neglect those of almost imperceptible height but from 600 to 1000 feet in length, which greatly influence the rolling movement of a ship. On the southern coast of Australia there is a well-known and remarkable difference in this respect in the character of the swell from the eastward and the westward. The south-east and the south-west directions there extend over equally great stretches of ocean, but while the swell from the south-east is a short chopping sea, high and steep (usually 8 or 9 feet high and 150 feet long, or as 1 to 17), that from the westward is a long heavy roll, usually about the same height (8 or 9 feet) but 150 *yards* instead of feet in length, or as 1 : 151. What I would here attempt to show, or rather to suggest, is that the varying ratio of height to length signifies or rather represents none other than the process of increase or subsidence of waves, and that if we could follow a sea-wave from its genesis to actual extinction we should be able to observe it through all the various phases as to height and length which have been enumerated.

That a certain force of wind acting for a given time will produce a wave of definite form is, I suppose, undoubted; and I presume it will not be questioned that the same conditions will always produce the precisely similar wave whose height is in a given ratio to its length. A certain force of wind, again, sufficient to obviate the loss by friction, will sustain in it this form; but if the sustaining force be withdrawn, then, however far its momentum will carry it—and it is known to carry it thousands of miles—the wave must thence gradually decline; and it is in this decline, viz., from its maximum height to final disappearance or extinction, that the ratio of height and length must, in this view, vary through all the degrees observed in waves.

But what is meant by the decline or the subsidence of a wave since the actual bulk or magnitude is neither measurable by its height nor its length, but by the area of a cross section? A volume of water has been raised to a certain height above the ordinary level; and in declining its height must decrease until the curve of its profile gets flattened out to a straight line. In what manner is the length thereby affected? Inquiry will, I think, show that the length is not only *relatively* increased (which it would be by remaining constant while the height alone decreased), but itself increases—that is, absolutely, in the act of the wave's subsidence.

Now, although we cannot accompany a wave in its onward progress across a sea to note the changes it undergoes in its transit, yet be it remembered that the same laws which influence deep-sea waves, however vast, likewise direct the movement of the smallest ripples, scanning which the eye may under favourable circumstances take in at a glance the phenomena here indicated.

For instance, if a fresh breeze be blowing on a small piece of water so as to produce a series of riplets, and these travel into a part which is sheltered from the wind, it will be observed that at genesis the wave is steepest—*i.e.*, the ratio of the length to the height small, and that as long as the wind has a direct active influence in sustaining them the height preserves a large proportion to the length. As soon, however, as the direct support of the wind ceases the wave begins to decline by, be it observed, spreading out in length and decreasing in height. The annexed diagram is made from observation in a spot favourably situated. The genesis of the riplet is at A (Fig. 1); from A to B, the point of maturity, it increases in size, the ratio of height to length being greatest during increase. In the mature stage (from B to C) the same ratio is maintained. At C, however, the wind has ceased its support, and thence to D the wave gradually subsides to extinction—*i.e.*, until the height becomes indefinitely small, and the length indefinitely great—in other words, the surface becomes flat.

Such a diagram may be said to represent the life of a sea wave in miniature, for although it is the *fac-simile* of the progress of a ripple only, from birth to extinction, the same reasoning obviously extends to that of the heaviest sea. For, be it observed, the largest sea must have had its origin in a primary wavelet, as at the point A; and we have only to extend the period of increase from A to B further towards D, as in the annexed figure (Fig. 2), to obtain the larger waves. The magnitude of the wave, in fact, is proportional to the period of increase, while being increasingly urged by the wind during the progress of the wave from A to B, and this time must obviously be dependent upon the extent of the *fetch* of free water over which the wind may extend; so that the strength and range of the wind being the same, the magnitude is proportioned to the fetch. A storm-wave therefore of forty feet in height may have the same profile as a ripple, from which indeed it must have sprung, and in the same way the declining ground-swell of an ocean has its miniature *fac-simile* in a pond.

The annexed diagram (Fig. 3) may practically illustrate the foregoing remarks. A represents accurately the average profile of the permanent south-west swell in the Southern Ocean in latitude from 40° to 48° S., arising from the prevailing winds around the Pole. The curve is taken from entries of a number of profiles drawn from observation in a recent voyage of the ship "Newcastle" from Melbourne *viâ* Cape Horn to London, and the same curve and dimensions are identifiable throughout in the same latitudes. B in like manner represents the profile and dimensions drawn to the same scale of presumably the same permanent south-west ground swell as it reaches the southern coast line of Australia, averaged from many sketches of such profiles taken on the spot. The outline A therefore represents the swell in its active or mature state at or about its maximum ratio of height to length in a stage when the height and the bulk of water moved oppress the mind with a sense of sublimity; and B represents it in its decline, when, after having traversed forty degrees of a great circle, or more than two thousand miles, it approaches dissolution. The height here is comparatively nil, and the length has increased almost to flatness. Yet this enormous swell had its origin in the Polar sea, as an initial wavelet, the relative magnitude of which could only be represented in the diagram by a dot.

Instead, however, of tracking a wave through this vast distance we may picture it as fixed and subsiding in a single spot without interfering with the logical sequence of the argument, inasmuch as it thus represents the same wave, filled by the same instead of by changed particles of the liquid to which its embodiment has been transferred.

Let, therefore (Fig. 4), *a, b, c, d, e* represent the profile of a wave from trough to trough, the dotted line *f, g* being the mean or smooth water level. So far as the subsidence is concerned we may wholly disregard the actual movements of the particles, and conceive an indefinitely thin layer of the liquid to be instantaneously fixed or congealed in the shape of the wave *a, b, c, d, e*. Here *i, c* is the height, and *a, i, e* the length of the wave.

It will be seen that the area *b, c, d, h* is that portion of the liquid which has been raised above the mean level of the ocean; while the areas *d, g, e* and *b, a, f* are that of the water which has been *thereby* depressed below the mean level; whence the area *b, c, d, h* above the mean level is equal to the sum of the areas *d, g, e,* and *b, f, a* below the mean level, since the filling of the lower areas by the upper would render the surface flat.

Conceive now that the rigidity is slackened, so that the ideal lamina becomes semi-viscous. The onward velocity of a wave keeps it from sinking suddenly, as does that of a hoop or a top; its decline, therefore, is not due to its onward velocity, and the slow sinking of a semi-viscous fluid may justly represent the process of its actual subsidence.

Taking this view then to be correct, we may, under such an assumption, consider the wave as wholly divested, not only of any onward motion, but also of any rotatory movement of the particles. This is nothing more than conceiving the form of the wave to be embodied of the same particles instead of successive ones.

If the sinking of the upper area merely filled up the lower areas, the length of the wave would still remain the same, viz., *a, i, e;* but observation shows that the length absolutely increases. Let the height of the wave have subsided to c', then instead of the profile being the curve f'', b'', c' d'', e'', which it would be if the length remained unchanged, it is represented by the curve a', b', c', d' e', whose length (the dotted line e', i, a') exceeds *e, i, a;* i', c' now represents the height of the declining form.

Now, in order to simplify matters, we may—the two halves of the wave being symmetrical—treat only of the half shapes, viz., *i, c, d, e,* and *i′, c′, d′ e′*. The area *c′, d′, h* is now equal to area *d′, e′, g′*, and *i′, c′* is the reduced height of the wave, the reduction taking place from both sides of the mean level. The actual quantity of subsidence is measured by the difference between the areas *e, i, c, d* and *e′, i′, c′, d′*, or perhaps by the difference of *d, h, c* and *d′, h, c′*, the change which occurs while the lamina of semi-viscous fluid is sinking into flatness. Whilst the exact expression for the profile curve is undecided—and it is to the determination of this that every inquiry on this subject at present tends—I am not aware so far as my own imperfect knowledge extends of any means of stating such difference: that is, of expressing the actual change in the ratio of height to length in precise terms of the diminution in height (viz., *i, c* —*i′, c′*).

But whatever be the precise function mathematically, the cause suggested will, I think, sufficiently account for all observed circumstances; and it will explain also the peculiar difference noted between the easterly and westerly swell on the southern Australian coast in respect of the ratio of height to length. In those parts the south-east winds are known to extend only, and therefore to act on the swell only, a few hundred miles from the shore; the waves therefore having their genesis within this distance have not space to reach a lengthy decline, or, perhaps, even full maturity. Whereas the south-west winds start from the Pole, and the swell arising therefrom has an unbroken fetch for attainment of the highest possible magnitude, and thousands of miles for the slow process of decline in which it gradually increases its length and diminishes its height. The westerly swell therefore reaches the Australian shore in its declining stage, when the length is great and the height small; the easterly, in its mature or steep stage, when the waves are therefore higher, shorter, and more active, being urged or having been more recently urged by the wind.

By the fetch of a particular wave at any moment is, of course, meant the distance it has travelled from its genesis as an initial wavelet until then. Let A (Fig. 5) be the point of commencement of the wave (and thence in most cases of the wind also), and A B its path or fetch when it is at the point B. If from points *a, b, c,* &c., in the fetch ordinates be erected representing the strength or velocity of the wind when the wave was passing those respective

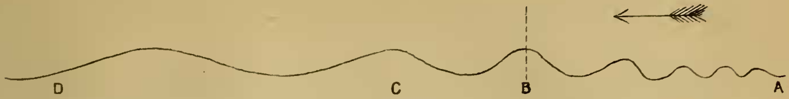
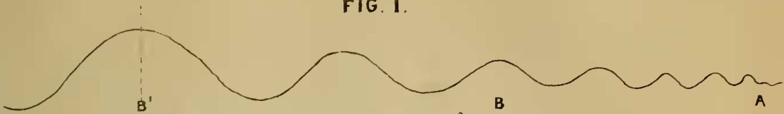
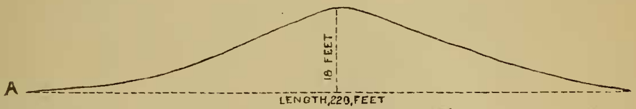


FIG. I.

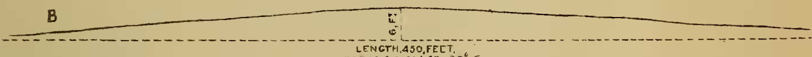


MAGNITUDE AT GENESIS LAT. 75°. S.

FIG. 2.



FORM AND MAGNITUDE IN LATITUDE. 48°. S.



FORM &c IN LAT. 36°. S,

FIG. 3.

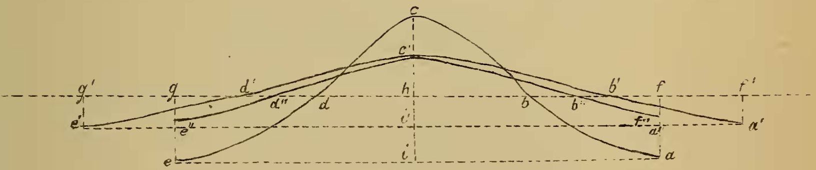


FIG. 4.

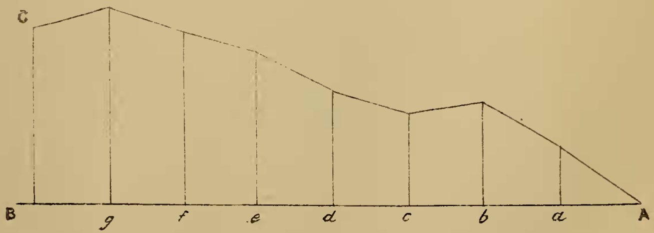


FIG. 5.